Department of Mechanical Engineering

ME4 Individual Project

'Topology Optimisation of Lattice Structure Additive Manufactured Knee Implants'

Seminar

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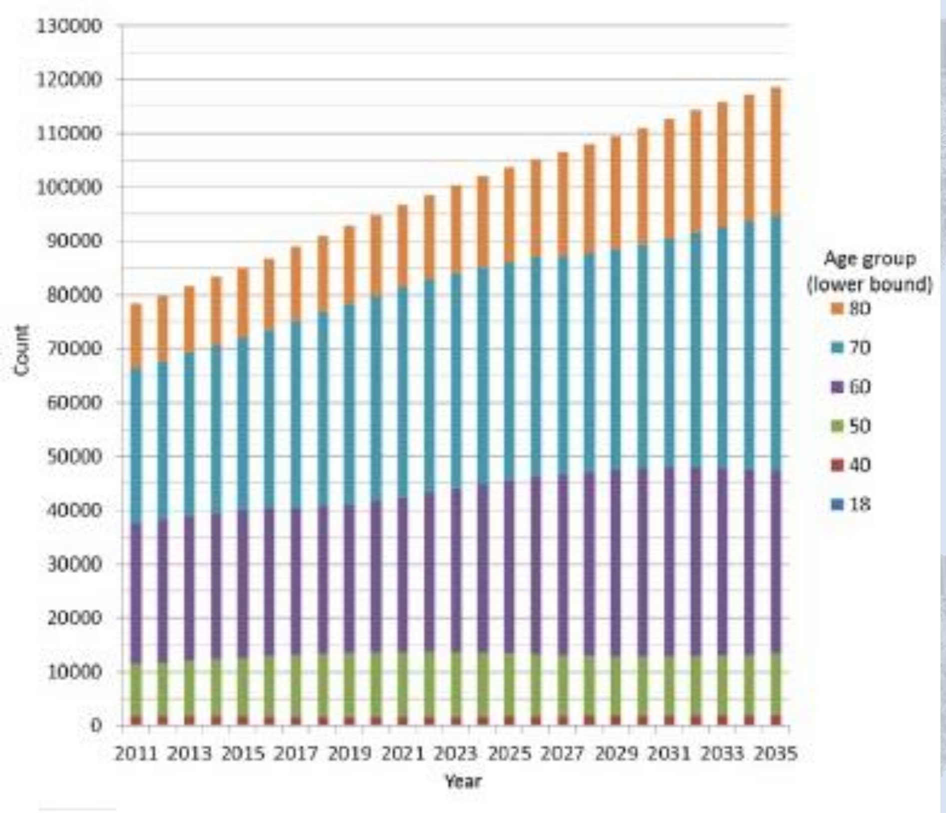
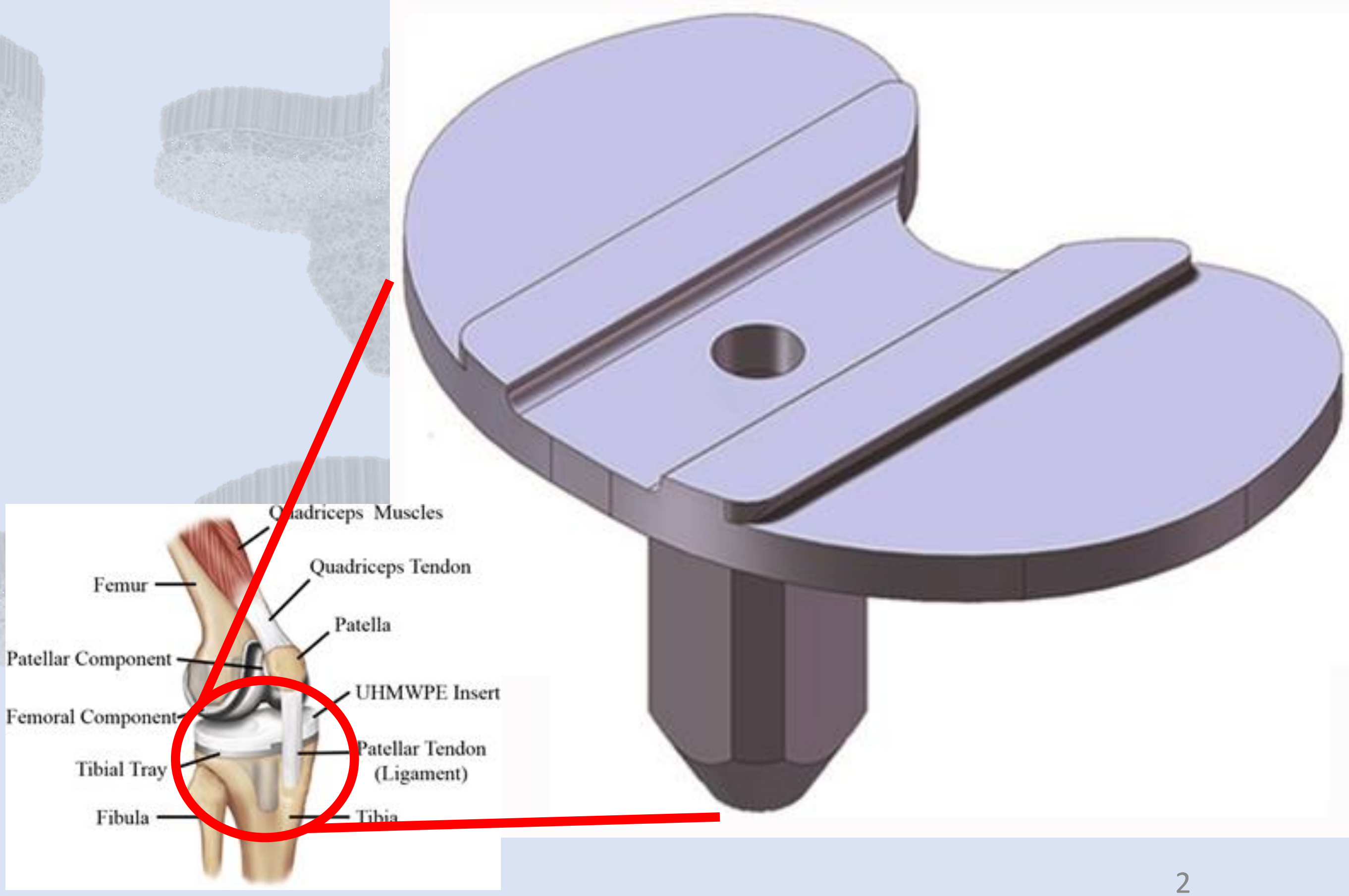
Date: 09/06/21

Introduction and Background

- Rising demand for knee implants.

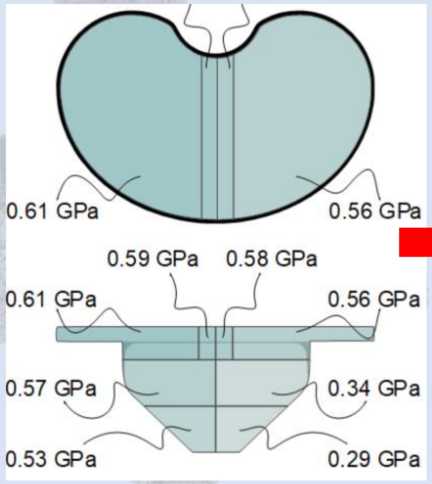


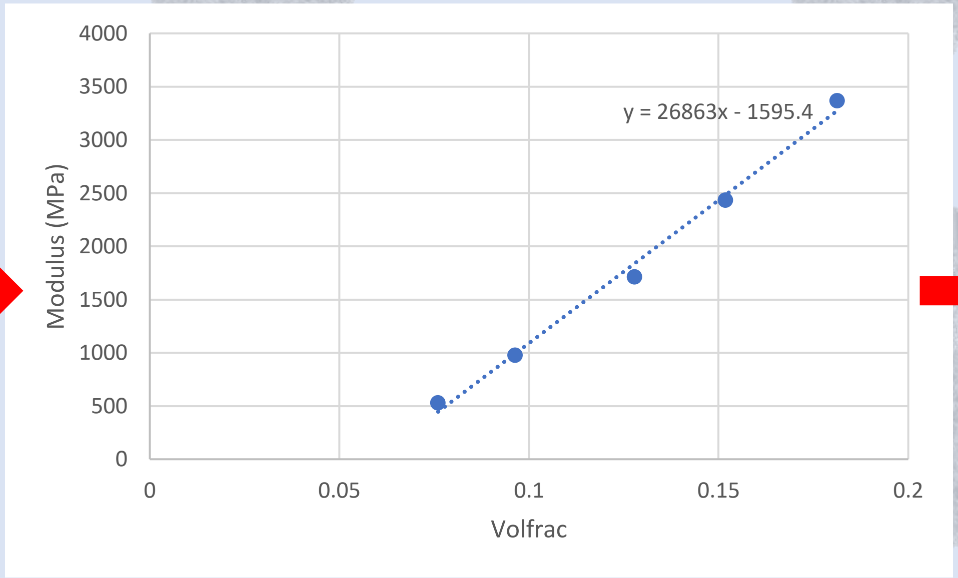
- Tibial tray component of knee implant resurfaces tibia.



Introduction and Background

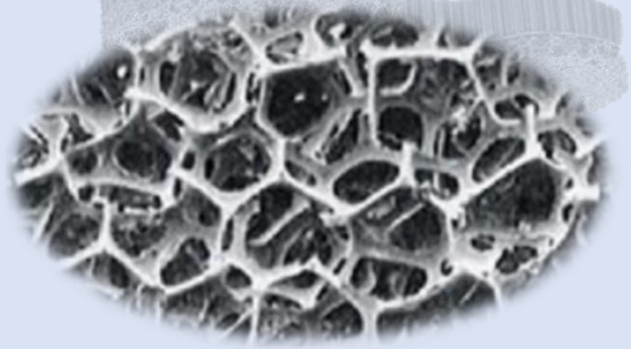
- Standard modern knee implants (~110 Gpa) have far greater modulus than surrounding bone → **stress shielding** effect.
- This **impedes effective bone regeneration** and leads to bone weakening → **implant loosening** and **complications**.
- The Biomechanics Group are developing a lattice structure knee implant.
- By establishing a linear relation, the modulus targets could be translated into lattice **volume fraction** targets.



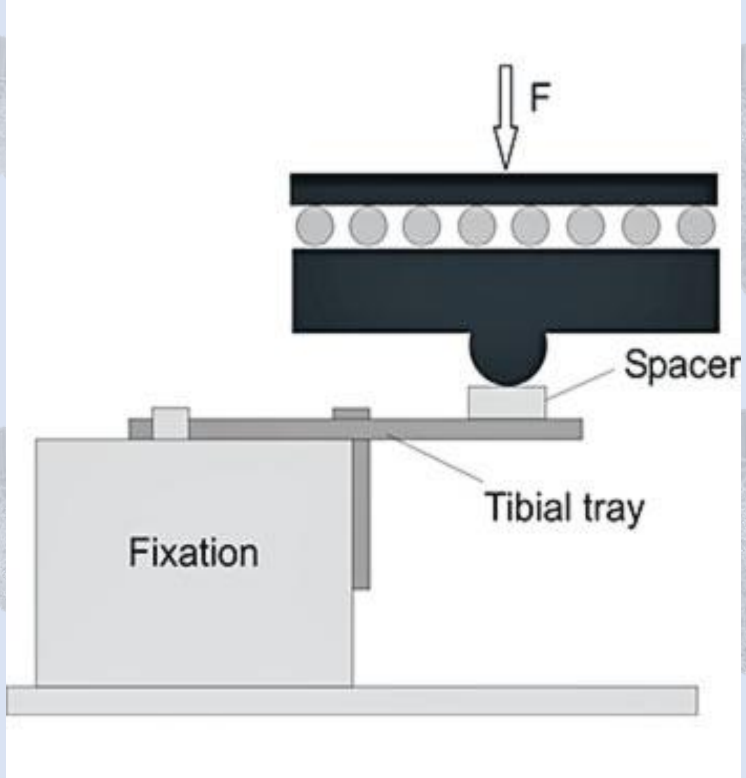


Region	Modulus Target (GPa)	Volume Fraction Target
Left 1	0.56	**0.080**
Left 2	0.34	**0.072**
Left 3	0.29	**0.070**
Right 1	0.61	**0.082**
Right 2	0.57	**0.081**
Right 3	0.53	**0.079**

Introduction and Background



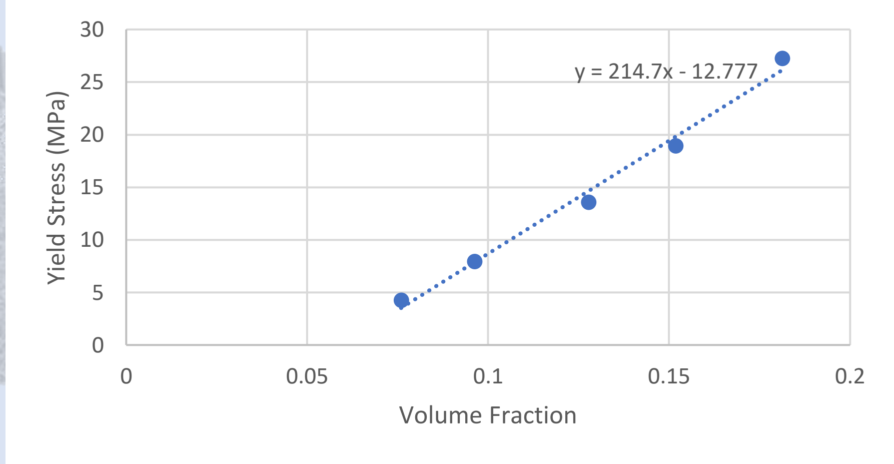
- Fully modulus matched tibial tray will not be strong enough to pass the **ISO load cycling test** which defines fatigue requirement for clinical use.
- To pass, 5 specimens must survive 10 million cycles of a 900 N load:



- Predicting lattice fatigue behaviour by S-N curve is complex:

$$S^* = C_A A_s \left(\frac{\rho^*}{\rho_s}\right)^{n_A} N_f^{*\,C_b b_s}$$

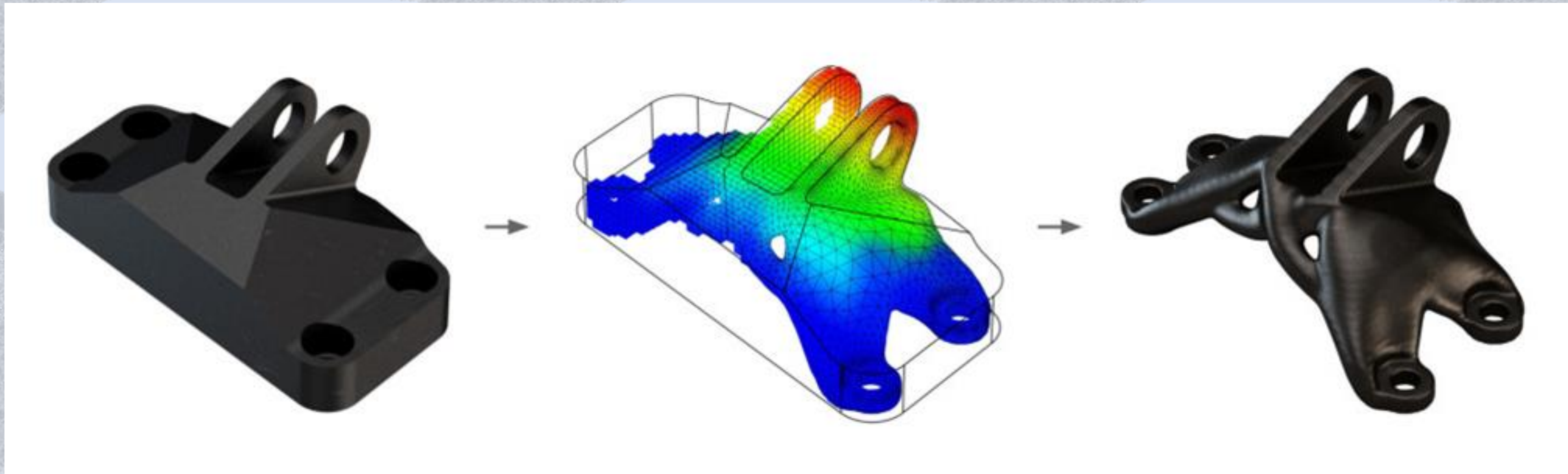
- Therefore focus was on **yielding-prevention** under ISO test's static 900 N load instead, by considering empirical volume fraction-yield stress relation.



Project Aim and Overview

Aim: To adapt the design of the lattice structure knee implant to improve yielding behaviour when exposed to a static ISO test load, whilst maintaining the favourable bone remodelling properties.

- This would make use of the **topology optimisation** approach.
- Typically applied to **binary compliance problems** via a **Solid Isotropic Material Penalisation** approach.



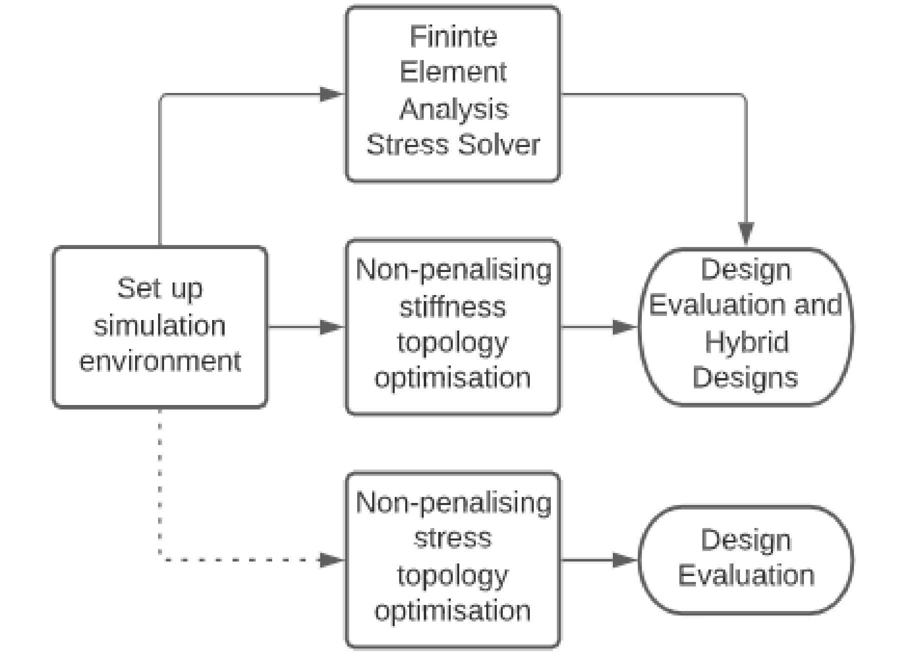
Project Aim and Overview

- Three main approaches were taken to the project aim, with resulting designs compared to the fully modulus-matched *'control'* design:

1. Developing a non-SIMP stiffness maximising topology optimisation with regional average volume fraction constraints

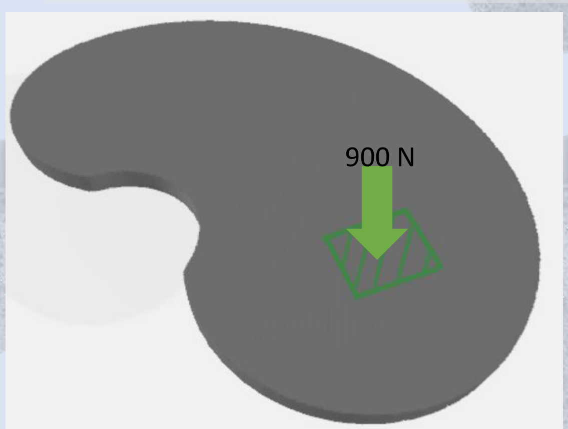
2. Developing a looped stress limiting process

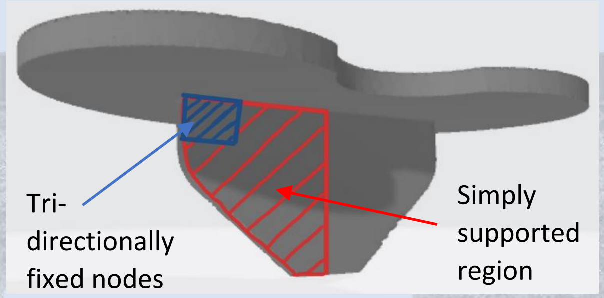
3. Developing a non-SIMP stress constrained topology optimisation

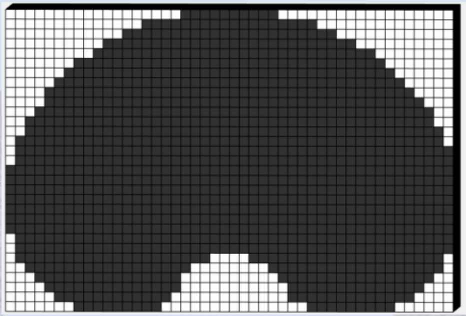


Simulation Environment and Validation

- Implemented **meshing** of CAD model in MATLAB and application of **loading and fixing condition**.







- Created structure **global stiffness and von mises stress matrices** through finite element methods.

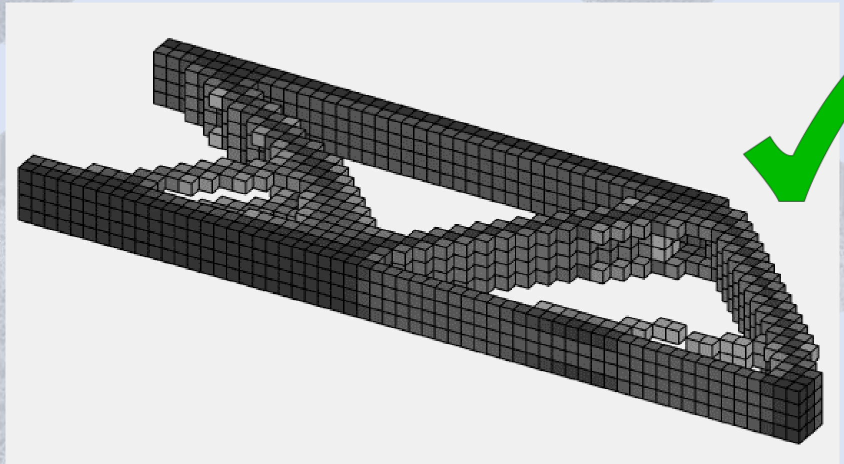
$$K(\rho) = \sum_{e=1}^{n} E_e \cdot K_e^0 \,;\; [K_e^0] = \int B^T C^0 B \; dVol$$

$$\sigma_e^{VM} = \left(\sigma_{e_x}^2 + \sigma_{e_y}^2 + \sigma_{a_z}^2 - \sigma_{e_x}\sigma_{e_y} - \sigma_{e_y}\sigma_{e_z} - \sigma_{e_z}\sigma_{e_x} + 3\sigma_{e_{xy}}^2 + 3\sigma_{e_{yz}}^2 + 3\sigma_{e_{xz}}^2\right)^{\frac{1}{2}} ;\; \sigma^e = [C^0][B][u_e]E_e = \begin{bmatrix} \sigma_{e_x} \\ \sigma_{e_y} \\ \sigma_{e_z} \\ \sigma_{e_{xy}} \\ \sigma_{e_{xz}} \\ \sigma_{e_{yz}} \end{bmatrix}$$

- Also implemented **adjoint method optimality criteria algorithm** for stiffness maximising topology optimisation.

Simulation Environment and Validation

- Running SIMP stiffness maximising topology optimisation on a cantilever beam CAD model with appropriate loading and fixing condition **reproduced standard result**:



Developed non-SIMP stiffness maximising topology optimisation with regional average volume fraction targets

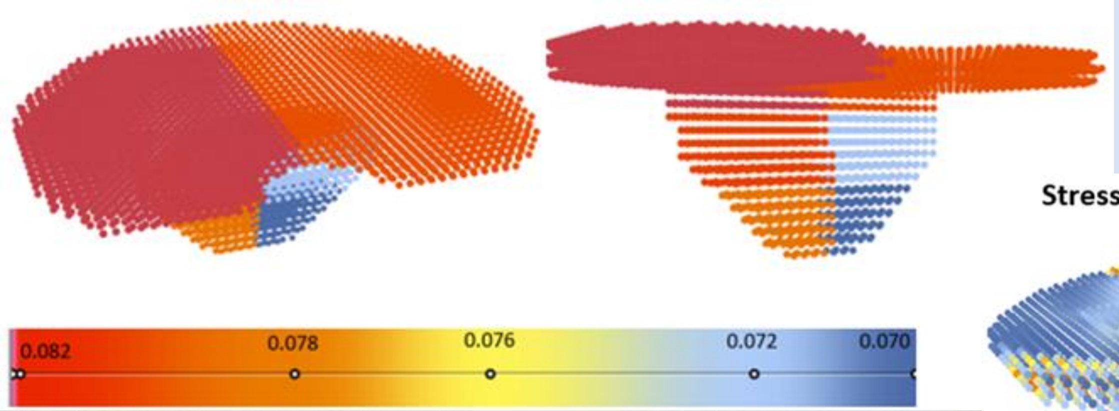
- Comparing simulation results for built-in stress of beam and end displacement to hand-calculated values showed similarity to an acceptable error:

	Hand Calc. Value	Value from Script	Error
End u, $F_N = 5N$	0.135 m	0.152 m	12.6%
Built in max σ_x, $F_N = 5N$	-1.13 MPa	-1.26 MPa	11.5%
Built in max σ_x, $F_N = 90N$	-20.25 MPa	-22.66 MPa	11.9%

Developed looped stress limited approach

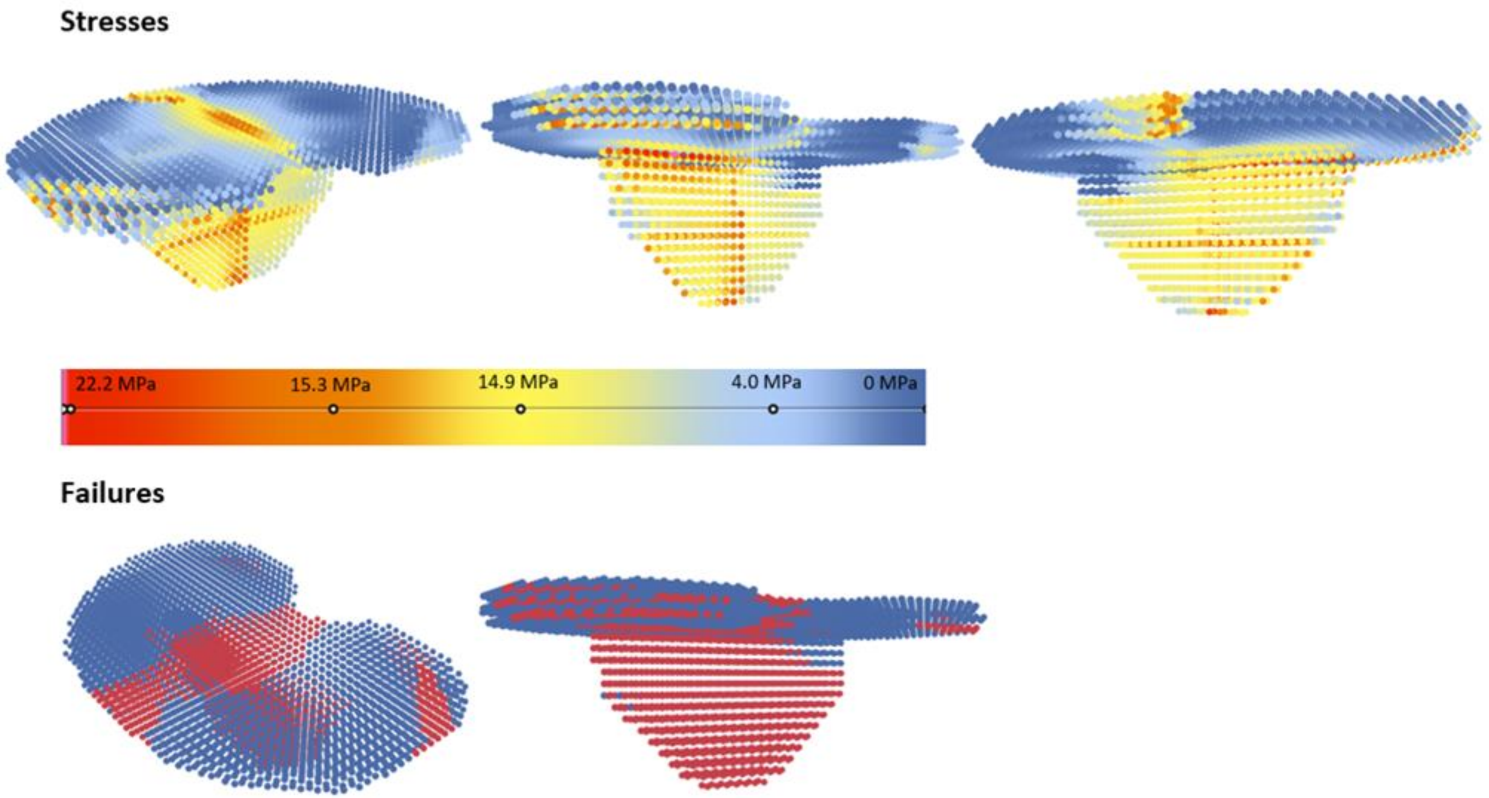
1

Fully Modulus-Matched *'Control'* Result



Volume fraction distribution

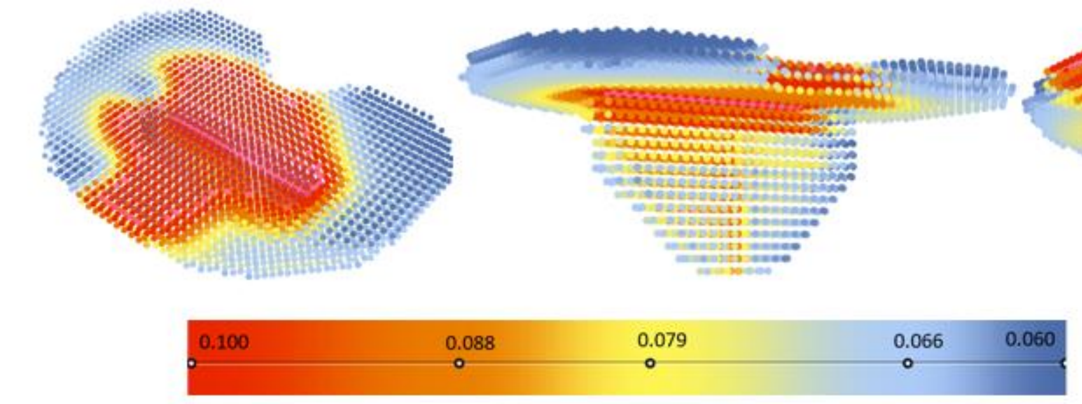
- Maximum stress:
  22.2 MPa
- Percentage predicted element failures:
  40.3%
- Overall average volume fraction:
  0.08
- Percentage average deviation from modulus targets:
  0%



Stress and failure distribution

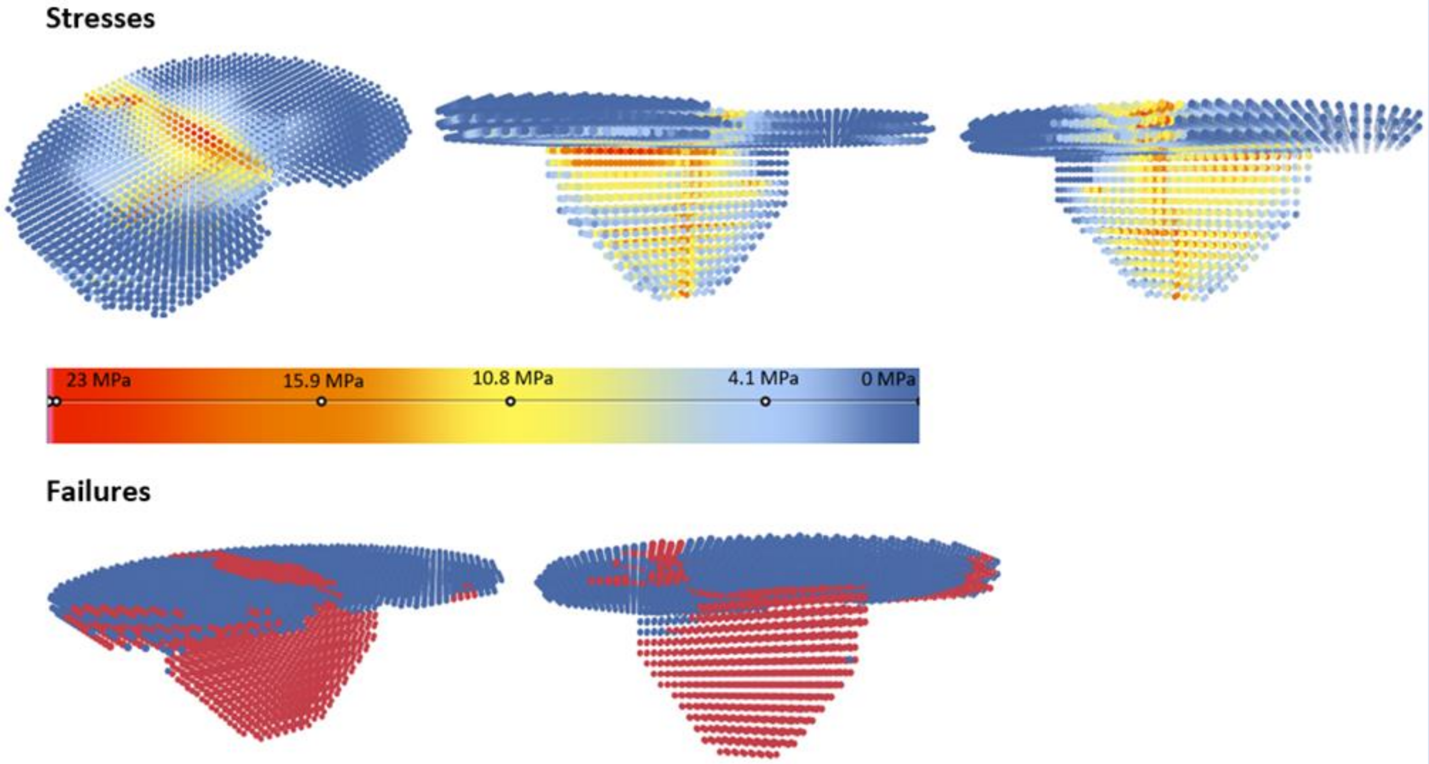
2

Non-SIMP Stiffness Maximising *'Stiffness Top Op'* Result



Volume fraction distribution

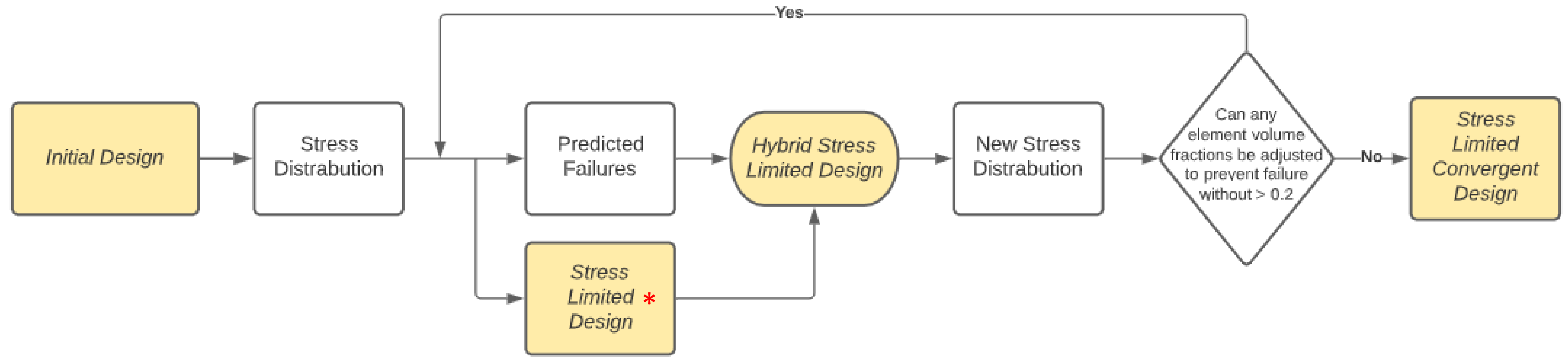
- Maximum stress:
  23.1 MPa
- Percentage predicted element failures:
  33.5%
- Overall average volume fraction:
  0.08
- Percentage average deviation from modulus targets:
  14.3%



Stress and failure distribution

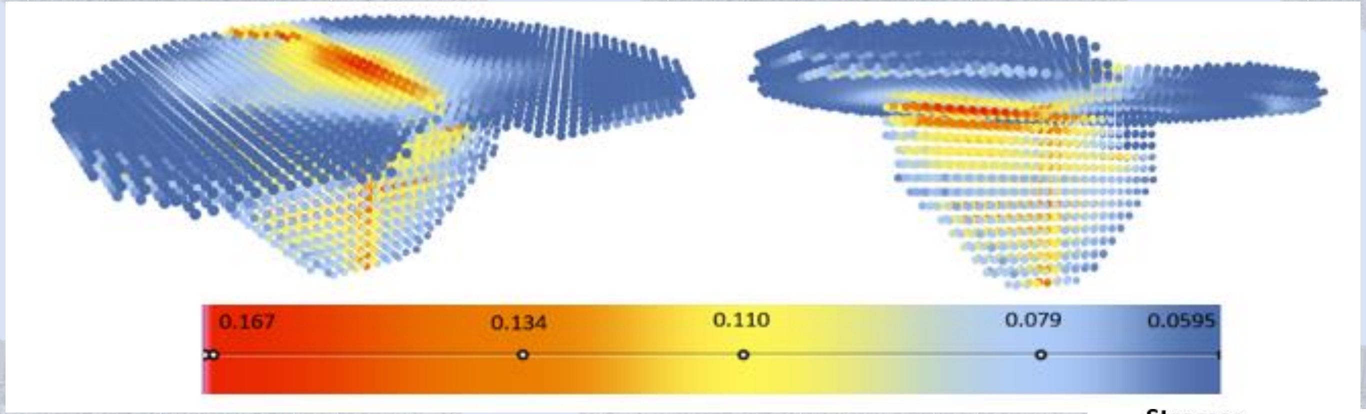
Stress Limiting Process

- **Convergence criteria**: no yielding subject to 0.2 volume fraction not being exceeded (limit set by additive manufacture capability).
- Results varied with **initial condition** and **iteration number**.
- 4 designs obtained from this process, shown in following slides.



3

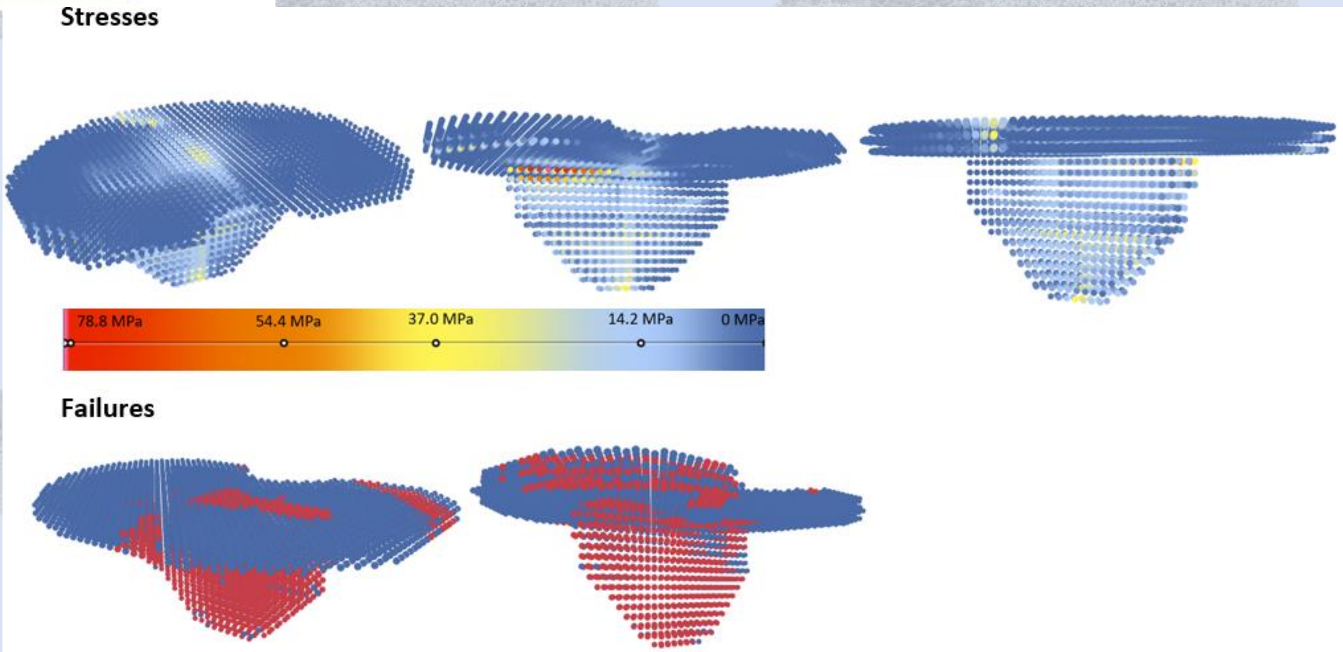
‘Stress Limited’ Result



Volume fraction distribution

‘Stress Limited’
- Initial condition: ‘*Stiffness Top Op’* design.
- Iteration no.: ‘half’ – all yield stresses matched to von mises stresses regardless of whether element failure predicted*.

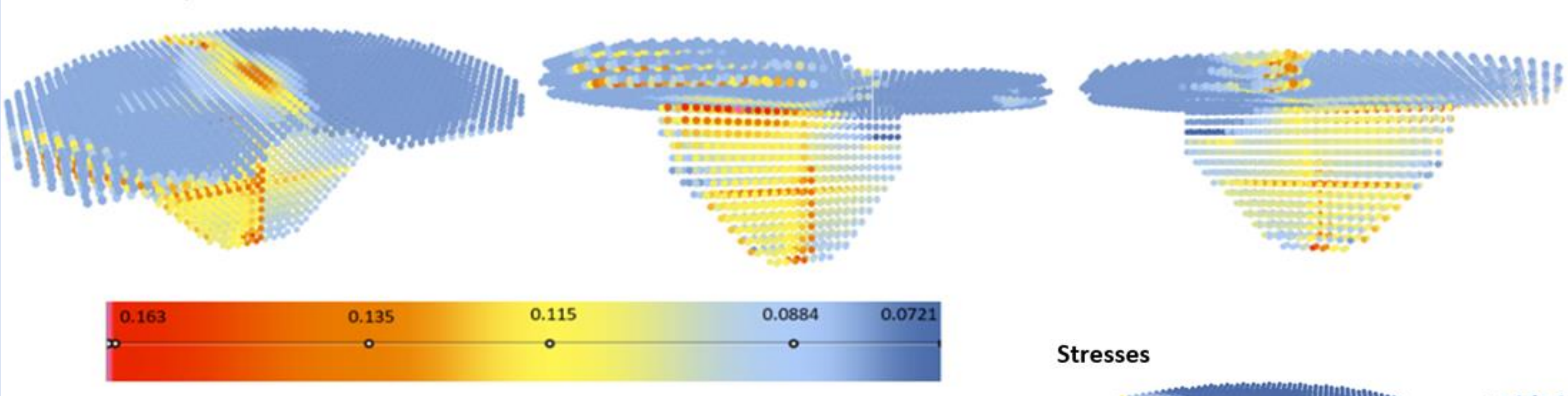
- Maximum stress:
  78.6 MPa
- Percentage predicted element failures:
  28.2%
- Overall average volume fraction:
  0.079
- Percentage average deviation from modulus targets:
  22.8%



Stress and failure distribution

4

'Stress Limited Control' Result

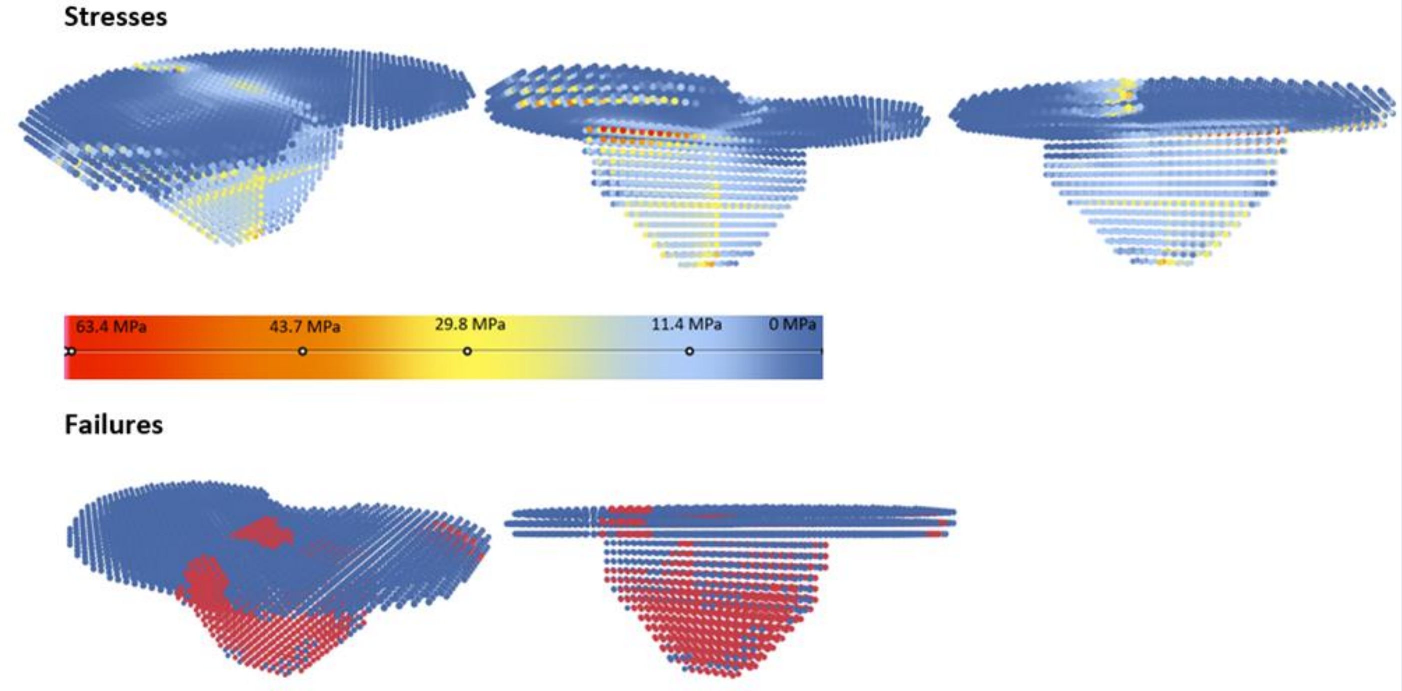


Volume fraction distribution

'Stress Limited Control'

- Initial condition: *'Control'* design.
- Iteration no.: single iteration.

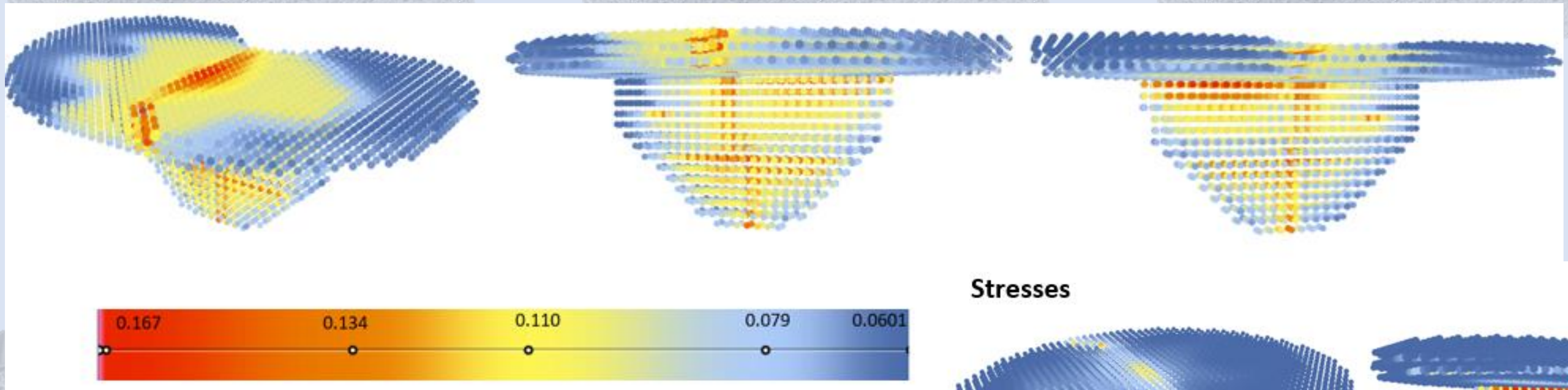
- Maximum stress:
  63.4 MPa
- Percentage predicted element failures:
  22.2%
- Overall average volume fraction:
  0.090
- Percentage average deviation from modulus targets:
  12.0%



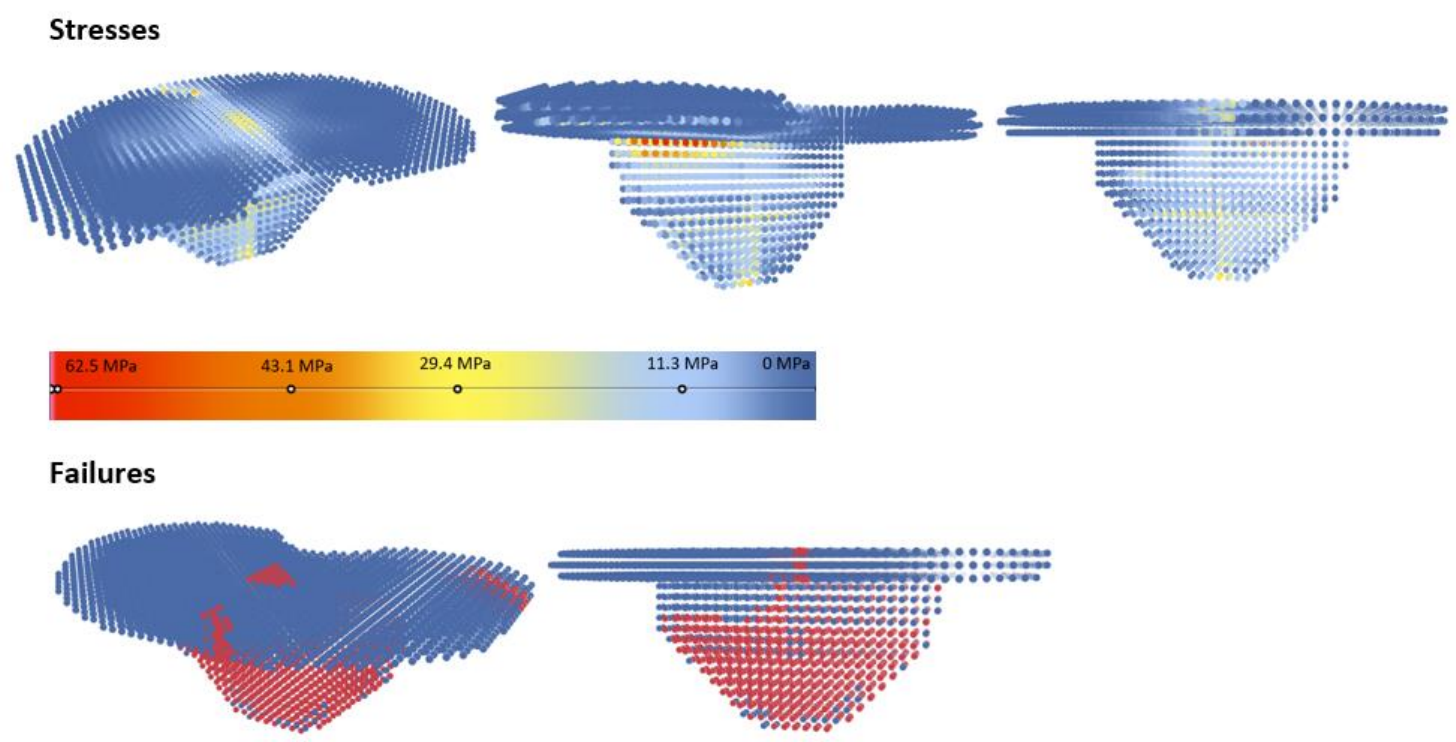
Stress and failure distribution

'Stress Limited Top Op' Result

'Stress Limited Top Op'
- Initial condition: *'Stiffness Top Op'* design.
- Iteration no.: single iteration.



Volume fraction distribution

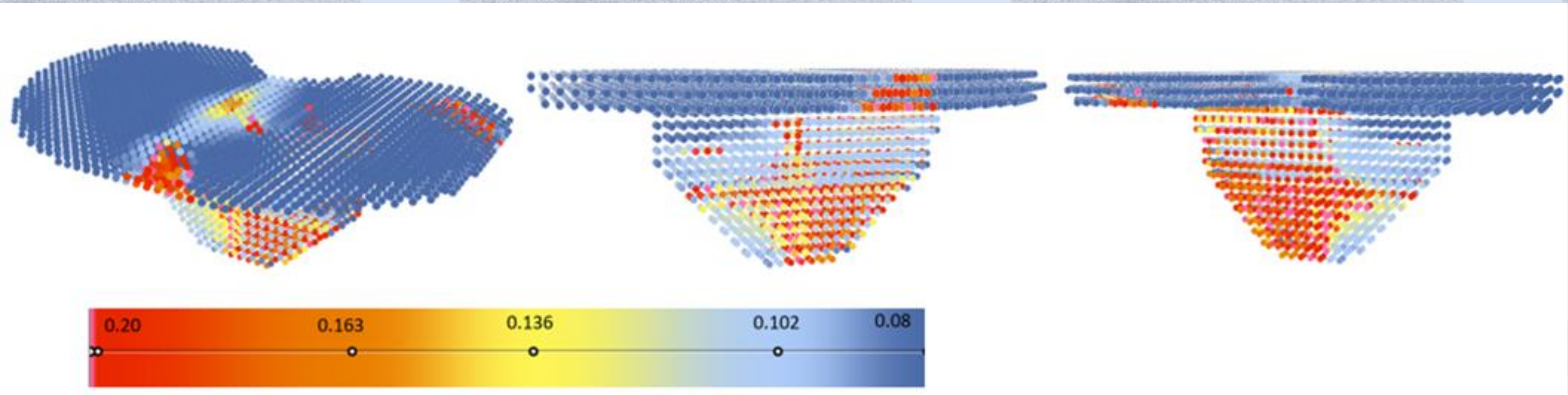


Stress and failure distribution

- Maximum stress:
  62.4 MPa
- Percentage predicted element failures:
  21.4%
- Overall average volume fraction:
  0.086
- Percentage average deviation from modulus targets:
  20.8%

6

'Convergent Stress Limited' Result

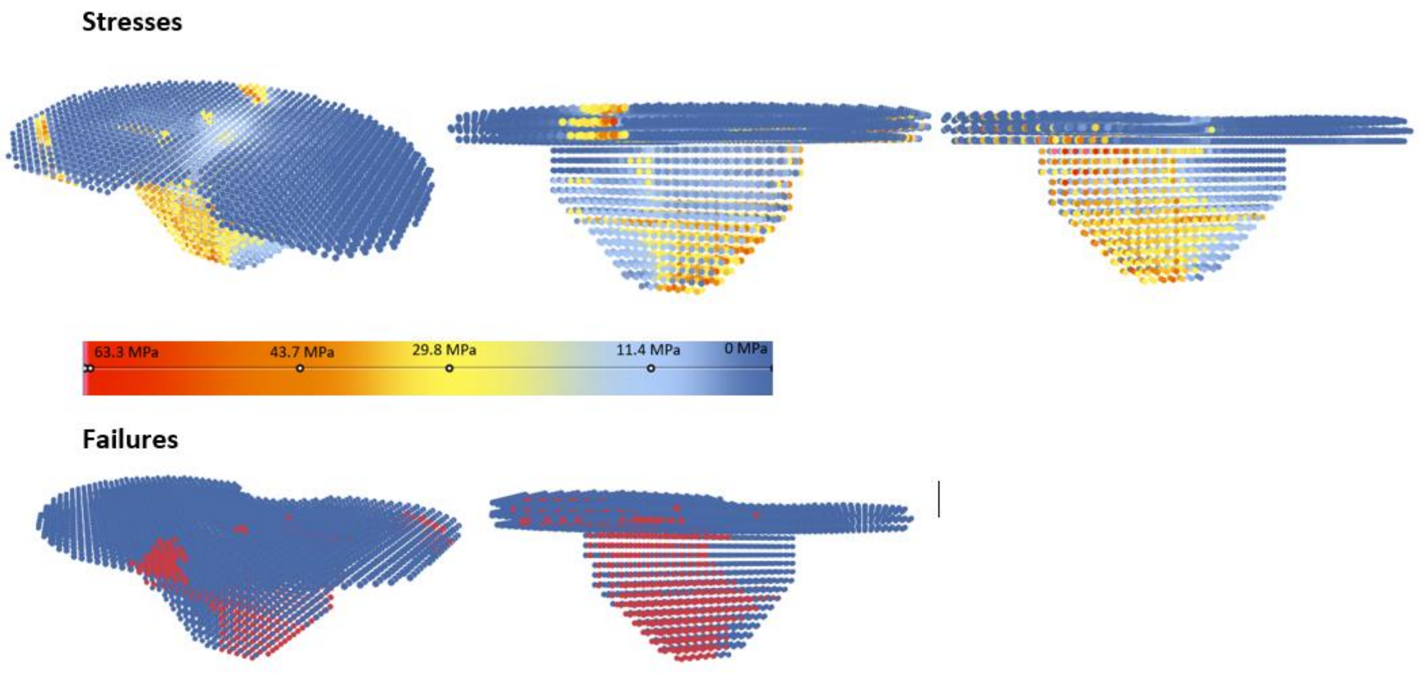


Volume fraction distribution

- Maximum stress:
  63.3 MPa
- Percentage predicted element failures:
  12.0%
- Overall average volume fraction:
  0.098
- Percentage average deviation from modulus targets:
  95.0%

'Convergent Stress Limited'

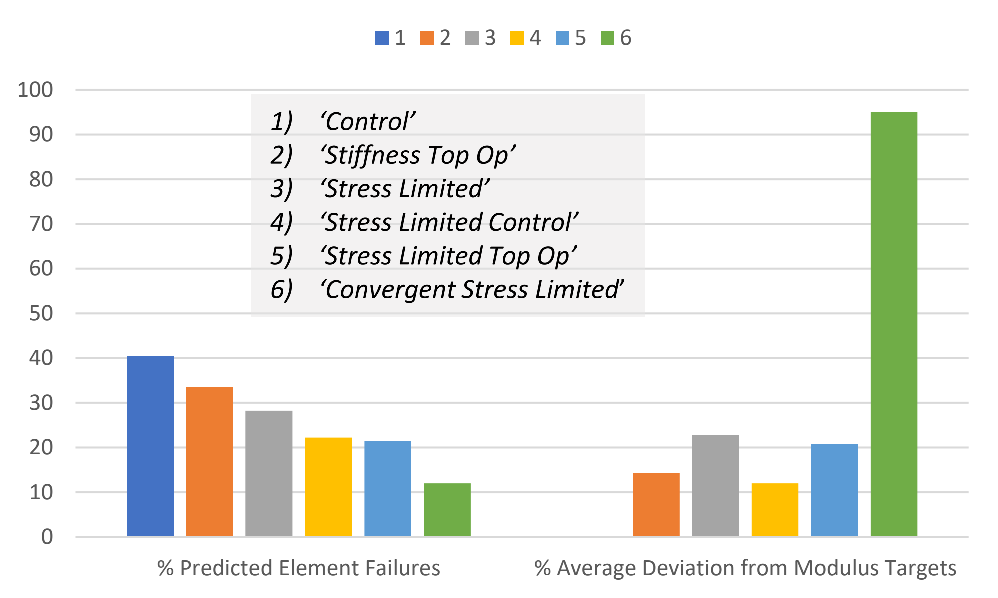
- Initial condition: all elements with 0.08 volume fraction (average overall target for bone regeneration).
- Iteration no.: until convergence

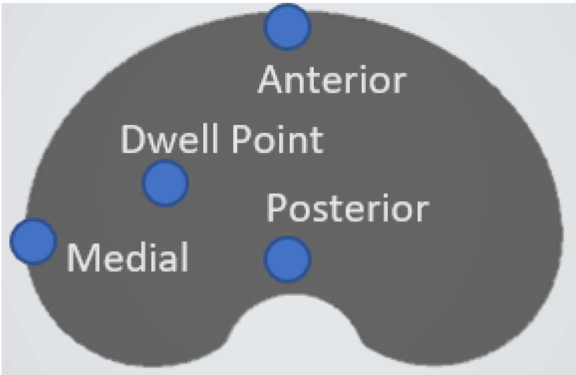


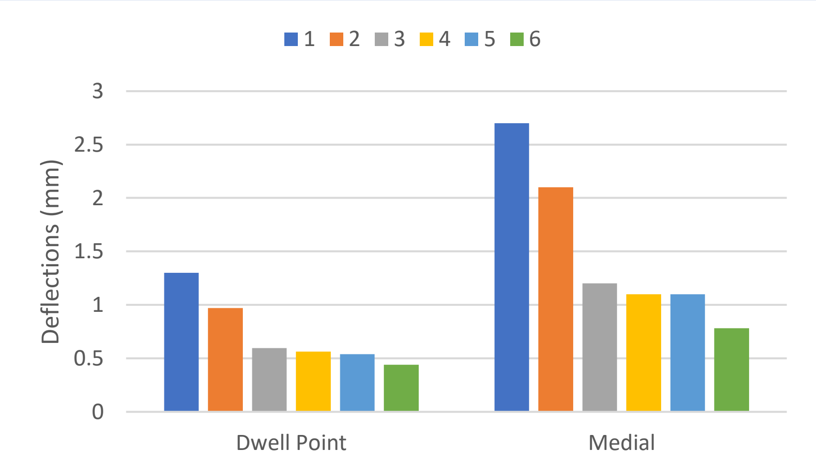
Stress and failure distribution

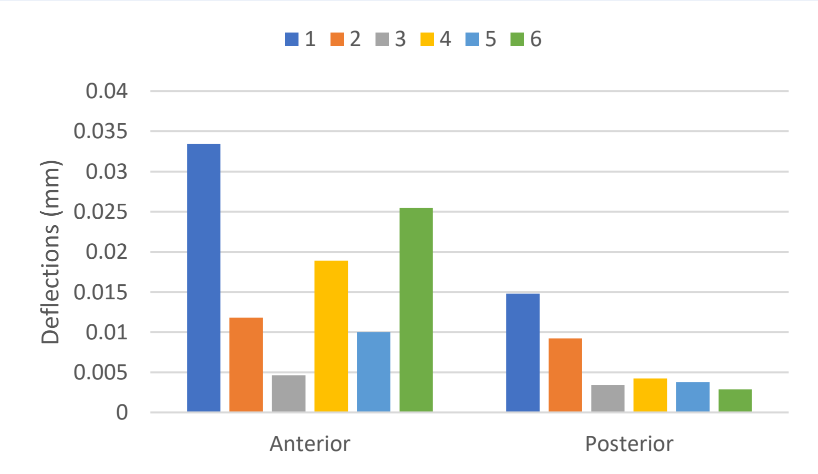
Discussion

- Consecutive designs **reduced deflections** at all indicated points (other than anterior) and **improves the failure behaviour**.
- Compromise upon bone remodelling behaviour doesn't increase in same order.
- Out of results obtained, *'Stress Limited Control'* performed best, with a 22.2% predicted element failure **(44.9% improvement upon '*Control*')** and a 12.0% modulus target deviation.









Stress Constrained Topology Optimisation

- A non-SIMP stress constrained topology optimisation which minimises the average volume fraction subject to stress constraints would be the most optimal solution.
- Optimisation required assembly of a global matrix of **P-norm stress** sensitivities with respect to each element's design variable for each cluster, which was derived analytically:

$$\frac{\partial \sigma_i^{PN}}{\partial x_b} = \sum_{a \epsilon \Omega_i}^{n_a} \frac{\partial \sigma_i^{PN}(x)}{\partial \sigma_a^{VM}} \left(\frac{\partial \sigma_a^{VM}(x)}{\partial \sigma_a} \right)^T \frac{\partial \sigma_a(x)}{\partial x_b}$$

P-norm stress: Elements grouped into 'clusters', and P-norm stress measure then approximates the maximum von mises stress of the cluster.

$$\frac{\partial \sigma_i^{PN}}{\partial x_b} = \sum_{a \epsilon \Omega_i}^{n_a} \left(\frac{1}{N_i} \sum_{a \epsilon \Omega_i}^{n_a} \left(\sigma_a^{VM}(x) \right)^p \right)^{\frac{1}{p}-1} \cdot \frac{1}{N_i} \left(\sigma_a^{VM}(x) \right)^{p-1} \cdot \begin{pmatrix} \frac{1}{2\sigma_a^{VM}} \left(2\sigma_{ax} - \sigma_{ay} - \sigma_{az} \right) \\ \frac{1}{2\sigma_a^{VM}} \left(2\sigma_{ay} - \sigma_{ax} - \sigma_{az} \right) \\ \frac{1}{2\sigma_a^{VM}} \left(2\sigma_{az} - \sigma_{ay} - \sigma_{ax} \right) \\ \frac{3}{\sigma^{VM}} \sigma_{axy} \\ \frac{3}{\sigma^{VM}} \sigma_{ayz} \\ \frac{3}{\sigma^{VM}} \sigma_{axz} \end{pmatrix}^T \cdot [C^0][B] \left(u_a \frac{\partial E}{\partial \rho} \cdot \frac{H(a,b)}{H_s(a)} + E_a \frac{\partial u_a}{\partial x_b} \right)$$

Analytical P-norm Stress Sensitivity Verification

- Attempted to verify analytical solution by finding results for the first 5 elements of a 3-cluster cantilever beam and **compare with finite difference approximations**.
- If sensitivities could be verified, would be able to run stress-constrained topology optimisation (given sufficient computational resource).

TABLE 1 – ANALYTICAL P-NORM SENSITIVITY VALUES

	w.r.t element 1	w.r.t. element 2	w.r.t element 3	w.r.t element 4	w.r.t element 5
Cluster 1	-8.4E+09	-8.9E+09	-1.3E+10	-1.4E+10	-1.6E+10
Cluster 2	-5.5E+07	-5.6E+07	-4.6E+07	-4.9E+07	-3.2E+07
Cluster 3	-7E+07	-7.5E+07	-5.5E+07	-1.1E+08	-3.8E+07

Table 2 – Finite difference P-norm sensitivity values with $\delta = 0.01$

	w.r.t element 1	w.r.t. element 2	w.r.t element 3	w.r.t element 4	w.r.t element 5
Cluster 1	-1.10E+07	-1.10E+07	-6860722	-6860722	-2645628
Cluster 2	-277265	-277265	-202156	-202156	-237028
Cluster 3	-97015.3	-97015.3	-85985.2	-85985.2	-247305

- Was not able to verify analytical solution – finite difference results all an order of 2-3 smaller. Future work should further investigate the complex P-norm stress sensitivity derivation.

Conclusions

- The **non-penalising stiffness-maximising topology optimisation** approach **improved yielding behaviour by 16.9% with a 14.3% deviation from modulus targets**.

- The **looped stress limiting approach** offered **further improvement in the yielding behaviour**, with degree of compromise upon modulus targets being **highly sensitive to initial condition and iteration number**.

- ***'Stress Limited Control'*** **design** resulted from a **single iteration upon a fully modulus-matched design** and **improved failure behaviour by 44.9% with only a 12.0% average deviation from modulus targets**.

Future work:

- Further explore and assess designs that can be made from stress limiting looped process.

- **Review P-norm stress sensitivity analytical solution** to validate successfully.

- **Lab testing** on additive manufactured designs to validate performance.